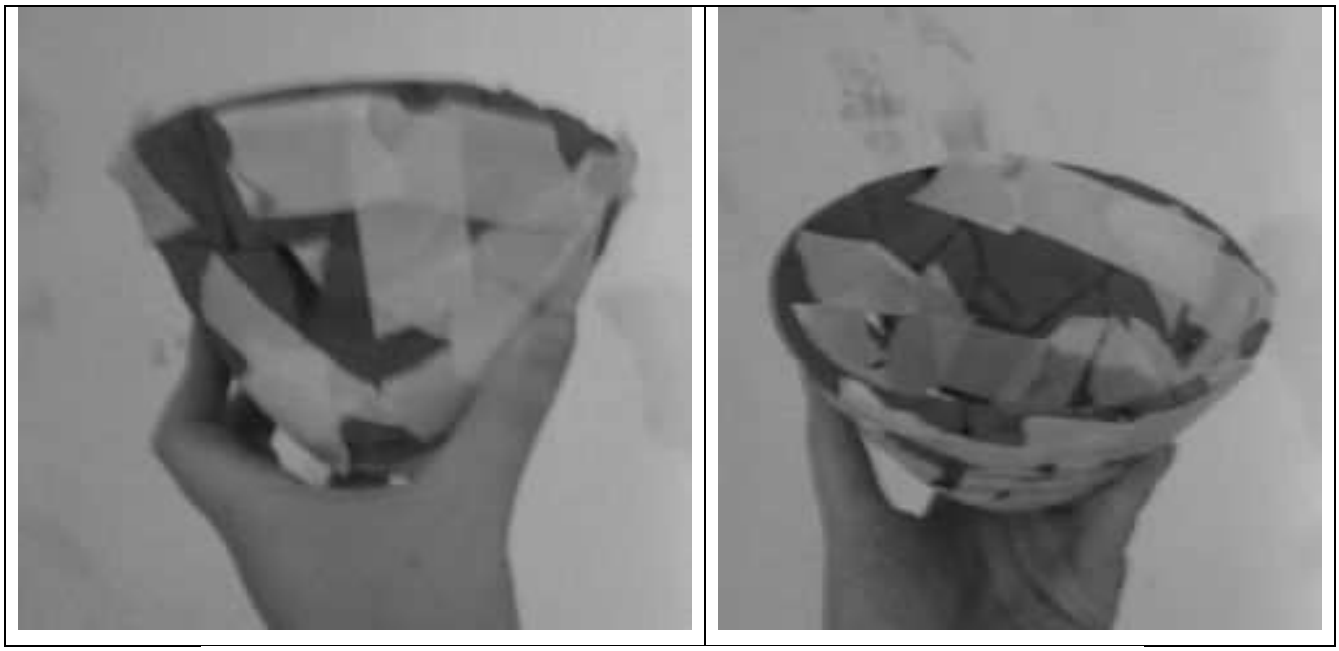
A few metrological remarks on a Twenty Second Dynasty collection of partially reconstructed containers at Mendes

In August 2007, the storage room of the Mendes dig-house contained a collection of nineteen Twenty-Second Dynasty partially restored containers (see sample below).



The restored containers unmarked and made of coarse reddish clay could not yield accurate volume readings. To decide which metrological units they represented and their possible role, the team tested their respective volumes with water and sand.

Data

To avoid problems caused by leaks, volunteers from the 2007 Expedition team carefully wrapped each container with a plastic film. They then performed two tests: one with water, the other with sand. Water provided a direct volume reading for most containers.

The weight of the construction sand used was 1.65 grams per milliliter. The team weighed each empty container, then filled each container with sand, and weighed it again. The difference in weight between each empty container and each container filled with sand divided by 1.65 (the weight of sand per milliliter) yielded the volume of each container. This method proved useful when leaks prevented a direct volume reading using water (table 1 below).

Table 1 – Volumes

Container Number	Reference	Shape	External Diameter (Ring Chart Reference Number)	Volume (Gross) water - ml	Weight - Sand grams (Gross)	Volume - Sand ratio 1.65 to ml
719	MP7-053	Conical	14	250	434.7	**263.45**
714	MP7-056	Conical	14	NA	434.7	**263.45**
720	MP7-059	Conical	14	250	434.7	**263.45**
740	MP7-055	Conical	15	270	447	**270.90**
780	MP7-052	Conical	13	345	579.6	**351.27**
727b	MP7-061	Conical	15	NA	579.6	**351.27**
76	MP7-100	Conical	15	NA	579.6	**351.27**
722	MP7-54	Conical	14	NA	579.6	**351.27**
711	MP7-057	Conical	17	438	724.5	**438.78**
7106	MP7-087	Conical	16	450	724.5	**439.15**
724	MP7-091	Conical	15	NA	724.6	**439.15**
746	MP7-063	Conical	17	NA	1014	**614.54**
751	MP7-077	Conical	20	NA	1152.9	**698.18**
748	MP7-064	Conical	18	825	1449.8	**878.66**
727a	MP7-036	Conical	23	NA	1449	**878.72**
736	MP7-058	Conical	22	650/700 ? (leak)	1449	**878.72**
7107	MP7-104	Conical	18	NA	1449	**878.72**
713	MP7-045	Conical	21	1000	1738.8	**1053.81**
725	MP7-60	Conical	22	NA	1738.8	**1053.81**

A ring chart, with increments of 0.5 centimeters, provided the external diameter as well as the internal radius of each container. The width of the brim of each container was 0.5 centimeter on average. Table 2 lists the internal radius of each container.

Table 2 – Radius

Container Number	Reference	Shape	External Diameter (Ring Chart) - cm	Internal Radius cm	Volume (Gross) water - ml	Weight - Sand grams (Gross)	Volume - Sand ratio 1.65 to ml
719	MP7-053	Conical	14	**6**	250	434.7	263.45
714	MP7-056	Conical	14	**6**	NA	434.7	263.45
720	MP7-059	Conical	14	**6**	250	434.7	263.45
740	MP7-055	Conical	15	**6.5**	270	447	270.90
780	MP7-052	Conical	13	**5.5**	345	579.6	351.27
727b	MP7-061	Conical	15	**6.5**	NA	579.6	351.27
76	MP7-100	Conical	15	**6.5**	NA	579.6	351.27
722	MP7-54	Conical	14	**6**	NA	579.6	351.27
711	MP7-057	Conical	17	**8**	438	724.5	438.78
7106	MP7-087	Conical	16	**7**	450	724.5	439.15
724	MP7-091	Conical	15	**6.5**	NA	724.6	439.15
746	MP7-063	Conical	17	**7.5**	NA	1014	614.54
751	MP7-077	Conical	20	**8.5**	NA	1152.9	698.18
748	MP7-064	Conical	18	**8**	825	1449.8	878.66
727a	MP7-036	Conical	23	**10**	NA	1449	878.72
736	MP7-058	Conical	22	**9.5**	650/700 ? (leak)	1449	878.72
7107	MP7-104	Conical	18	**8**	NA	1449	878.72
713	MP7-045	Conical	21	**9.5**	1000	1738.8	1053.81
725	MP7-60	Conical	22	**10**	NA	1738.8	1053.81

The volume of a cone equals one third of the volume of a cylinder of equal height and base (Lions:1980, p.212). The volume of a cylinder equals the area (*ʒḥt*) of its base (*tp-rʒ* [the diameter corresponds to *ḥr*]) multiplied by its height (*ʿḥʿ* [Lauer:1965, pp. 24-35 ; Struve:1930, pp. 160-1]). Its height is therefore its volume divided by the area of its base. Table 3 gives the theoretical internal height of each container.

Table 3 - Theoretical Internal Heights

Container Number	Reference	Volume - Sand ratio 1.65 to ml	Volume Cylinder Matching Height	Base Area	Theoretical Internal height (cm)
719	MP7-053	263.45	790.35	113.04	**6.99**
714	MP7-056	263.45	790.35	113.04	**6.99**
720	MP7-059	263.45	790.35	113.04	**6.99**
740	MP7-055	270.90	812.7	132.67	**6.13**
780	MP7-052	351.27	1053.81	94.99	**11.09**
727b	MP7-061	351.27	1053.81	132.67	**7.94**
76	MP7-100	351.27	1053.81	132.67	**7.94**
722	MP7-54	351.27	1053.81	113.04	**9.32**
711	MP7-057	438.78	1316.34	200.96	**6.55**
7106	MP7-087	439.15	1317.45	153.86	**8.56**
724	MP7-091	439.15	1317.45	132.67	**9.93**
746	MP7-063	614.54	1843.62	176.63	**10.44**
751	MP7-077	698.18	2094.54	226.87	**9.23**
748	MP7-064	878.66	2635.98	200.96	**13.12**
727a	MP7-036	878.72	2636.16	314.00	**8.40**
736	MP7-058	878.72	2636.16	283.39	**9.30**
7107	MP7-104	878.72	2636.16	200.96	**13.12**
713	MP7-045	1053.81	3161.43	283.39	**11.16**
725	MP7-60	1053.81	3161.43	314.00	**10.07**

Interpretation

The containers examined are considered to be drinking cups. A quick iconographic survey shows that drinking cups follow two basic shapes:

a/ the shape of half a ball (either spherical or elliptical [First Intermediate period: Aldred:1985 - p.110, Smith:1988, p.153]).

b/ the shape of the cup glyph Gardiner's sign list W10 (Gardiner:1988, p.528; Ramesside period: Hornung:1990 - p.65, Aldred:1985 - p.191).

The containers in the Mendes collection have a pronounced conical shape not usually found in the drinking-cup iconographic record. When held in one's hand their fit and conical shape turn them into a natural and perfect instrument to scoop grain from a larger container or a sack.

Their respective volumes match standard metrological units used from the Middle Kingdom onwards: a *hin* or a *dja*, the basic divisions of the *hekat*. Both units have commonly attested divisions and multiples (Pommerening:2005, pp.39-40): one half, two, and four found in the samples. There are examples of three and five *dja* units in the Late period (Pommerening:2005, p.260).

Table 4 - Metrological Classification

Container Number	Reference	Shape	External Diameter (Ring Chart) - cm	Volume (Gross) water - ml	Weight - Sand grams (Gross)	Volume - Sand ratio 1.65 to ml	Units
719	MP7-053	Conical	14	250	434.7	263.45	**1/2 *hin***
714	MP7-056	Conical	14	NA	434.7	263.45	**1/2 *hin***
720	MP7-059	Conical	14	250	434.7	263.45	**1/2 *hin***
740	MP7-055	Conical	15	270	447	270.90	**1/2 *hin***
780	MP7-052	Conical	13	345	579.6	351.27	**1 *dja***
727b	MP7-061	Conical	15	NA	579.6	351.27	**1 *dja***
76	MP7-100	Conical	15	NA	579.6	351.27	**1 *dja***
722	MP7-54	Conical	14	NA	579.6	351.27	**1 *dja***
711	MP7-057	Conical	17	438	724.5	438.78	**1 *hin***
7106	MP7-087	Conical	16	450	724.5	439.15	**1 *hin***
724	MP7-091	Conical	15	NA	724.6	439.15	**1 *hin***
746	MP7-063	Conical	17	NA	1014	614.54	**2 *djas***
751	MP7-077	Conical	20	NA	1152.9	698.18	**2 *djas***
727a	MP7-036	Conical	23	NA	1449	878.72	**3 *djas***
736	MP7-058	Conical	22	650/700 ? (leak)	1449	878.72	**3 *djas***
7107	MP7-104	Conical	18	NA	1449	878.72	**3 *djas***
748	MP7-064	Conical	18	825	1449.8	878.66	**3 *djas***
713	MP7-045	Conical	21	1000	1738.8	1053.81	**2 *hins***
725	MP7-60	Conical	22	NA	1738.8	1053.81	**2 *hins***

From the New Kingdom onwards the volume of a *dja* (300 to 333 ml) is one third of the volume of a *hin* (450 to 500 ml (Pommerening:2005, pp. 245-6). Both remain divisions of the *hekat*. These two units explain the origin of ancient Egyptian metrology based on two taxable commodities: emmer and barley. For an equal weight, the volume of a container of emmer will be bigger by one third than the volume of a container of barley. Scales are not necessary anymore, being replaced by units of volume. One liter of emmer weighs between 400 and 440 grams. One liter of barley weighs between 600 and 660 grams (Hirsch:2005, pp.36-68). One *dja* of barley (333 ml multiplied by .66 [the specific weight of barley]) weighs 219.78 grams and equals the weight of one *hin* of emmer, 220 grams (500 ml multiplied by .44 [the specific weight of emmer]).

Applied to the examples in the collection, the specific weights of emmer and barley yield the weight equivalence listed in table 5.

Table 5 - Samples Weight Equivalence

Commodity (1)	*Hin*	Volume (1) ml	Weight (1) grams	Weight (2) grams	*Dja*	Volume (2) ml	Commodity (2)
Emmer	1	500	**220**	**219.78**	1	333	Barley
Emmer	2	1000	**440**	**439.56**	3	999	Barley
Emmer	2	1000	**440**	**439.56**	3	999	Emmer
Barley	2	1000	**666**	**659.34**	3	999	Barley

It is interesting to note that nature (the weight characteristics of both emmer and barley) provide a practical weight standardization: the volume of two *hins* equal in weight the volume of three *djas* as far the two taxation commodities (emmer and barley) are concerned.

There are no metrological marks on the containers examined. They could be unofficial multipurpose household "china." Scooping does not exclude using them as drinking cups even if their original purpose was different.

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